MATHEMATICS D

Paper 4024/11
Paper 1

Key messages

To do well on this paper, candidates need to have covered the whole syllabus, accurately learned the necessary formulae, clearly show all the necessary working for each question, perform calculations accurately and give answers to an appropriate degree of accuracy.

General comments

The paper contained questions accessible to all candidates. Areas of the syllabus that need further attention include estimation, functions and geometrical problems involving circles and polygons.

It is clear from errors made on the paper how important it is to constantly practice basic non-calculator work, in particular multiplication tables.

Presentation of the work was generally good. It is important that all numbers written by candidates are clear – too often numbers are written hurriedly and are not formed correctly, particularly later on in the paper.

Working should not be deleted as it may contain something of merit if the correct answer is not obtained.

Candidates should always check answers, particularly in practical problems such as **Question 2** and **Question 4**, to see if the answer is sensible.

Always check that the answer obtained in the working space is correctly transferred to the answer line.

If a candidate thinks their initial answer is incorrect, this is best deleted and replaced rather than attempting to overwrite the answer.

Comments on specific questions

Question 1

(a) This part was answered well with most candidates initially converting the mixed numbers to top-heavy fractions.

(b) There was a high proportion of correct answers. Many candidates attempted either long multiplication (which sometimes resulted in incorrect placement of the decimal point) or conversion to fractions before multiplying, rather than simply multiplying 3 by 11 and counting up the decimal places.

Answers: **(a)** $\frac{17}{30}$ **(b)** 0.0033

Question 2

The contextual nature of the question proved difficult for many candidates.

(a) The initial step of dividing 450 cm by 60 cm was frequently correct but candidates did not always note the context of the question and left the answer as $7\frac{1}{2}$ (or 7.3 rather than $7\frac{3}{6}$), not realising that the number of pieces had to be a whole number, or rounded their answer up to 8 pieces.

(b) Candidates who gave the correct number of pieces in **part (a)** usually reached the correct answer. Those with $7\frac{1}{2}$ in **part (a)** often realised that whole pieces were required here and successfully gave an answer of 30 cm although some gave an answer of 0. A number of candidates did not attempt this part.

Answers: **(a)** 7 **(b)** 30

Question 3

(a) This part proved difficult for a significant number of candidates, with problems dealing with the fraction or decimal divided by 100. Basic arithmetic errors were often made in the cancelling and the fraction was sometimes cancelled too far , e.g. $\frac{1.3}{4}$. Weaker candidates often multiplied, rather than divided, the given percentage by100.

(b) This part was generally answered well.

Answers: **(a)** $\frac{13}{40}$ **(b)** $\frac{7}{20}, \frac{9}{25}$, 0.38, 0.4

Question 4

(a) Incorrect conversion of kilograms to grams led to answers greater than the cost of 1 kg such as \$48 and \$480. Candidates should be aware of the need to check whether their answer is sensible or not.

(b) This part was answered well.

Answers: **(a)** 4.80 **(b)** 24

Question 5

(a) There was a lack of understanding as to whether or not a 0 was a significant figure with common wrong answers of 360.000, 36 or 36.0000. Some gave 358 as the answer or gave the answer to two decimal places.

(b) The key to questions of this type is to recognise that estimation is asking for the numbers to be approximated before trying to carry out the calculation. Those who did this, giving the numerator and the denominator as being approximately four and one, were successful. Attempts were made to cube 67 (or divide it by three), rather than cube root it. This part was not attempted by a number of candidates.

Answers: **(a)** 360 **(b)** 4

Question 6

Most candidates read the question carefully enough to recognise a correct form for the inverse proportion formula and were mainly successful in finding $k = -150$. Problems were then encountered with either the negative sign for k or forgetting to square x, leading to some incorrect final answers for y. The incorrect formula $y = \frac{x}{k}$ was occasionally seen.

Answer: 15

Question 7

The calculation for this type of question is usually much easier, and more successful, if using the exterior angle result rather than that for the interior angle. Those candidates who correctly knew either formula usually then went on to give the correct final answer, although a number gave 4 as the answer to $\frac{360}{9}$. The common error was to use 171 = 180(n – 2).

Answer: 40

Question 8

(a) Those recognising that $2^0 = 1$ were successful. The common errors were to think $2^0 = 0$, to calculate $2^{3-0} = 2^3$ or to cancel the 2's and find 3 – 0 = 3.

(b) This was generally answered well. Errors included taking out a common factor of 3*x*, rather than cancelling by 3*x*.

Answers: **(a)** 7 **(b)** $\frac{4y}{3x}$

Question 9

(a) The common errors were to give the answer as 0.15, which was not accurate enough, or as $\frac{60}{4} = 15$. Lines drawn on the diagram horizontally from 15 on the cumulative frequency axis to the curve and then vertically down to the time axis could have assisted the accuracy of the answers given.

(b) The correct answer was often seen with the most common wrong answer being 40, as candidates found the number with a reaction time of less than (rather than more than) 0.2 seconds.

Answers: **(a)** 0.155 **(b)** 20

Question 10

(a) This part was well answered.

(b) Candidates who realised that the numbers needed to be divided and the powers subtracted were often able to reach 0.3×10^{10}, although some gave 10^{-10} or 10^0. Answers were then not always converted into standard form or given as 3×10^{11}. A few calculations involved 1.5×5.

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Answers: **(a)** 4.5×10^8 **(b)** 3×10^9

Question 11

(a) Although frequently correct, some answers were left as $\dfrac{3\frac{1}{2}}{10}$. The correct initial step of $\dfrac{(3-(-\frac{1}{2}))}{10}$ sometimes became $\dfrac{(3-\frac{1}{2})}{10} = 0.25$ and $\dfrac{7}{2} \div 10$ sometimes became $\dfrac{7}{2} \times \dfrac{10}{1}$. Some candidates took f(x), rather than x, as $-\dfrac{1}{2}$.

(b) Although there were many successful attempts, some candidates interchanged x and y without going on to rearrange their formula to give y in terms of x. Others rearranged first but forgot to interchange the variables or reached an answer of $10x - 3$ and a few found 1/f(x).

Answers: **(a)** 0.35 **(b)** $3 - 10x$

Question 12

(a) (i) A fair number of candidates gave the correct number of elements but others gave the answer as eight, listed the elements themselves or found the number of elements in the intersection of the two sets. The answer should be a number only, not n(9).

(ii) The common errors here were to give an answer of 91 or to give at least two values (often including 91), even though the question asked for the **value** of y.

(b) There were many correct answers with common errors being to shade all of B, shade A' or shade the intersection of A and B.

Answers: **(a)(i)** 9 **(ii)** 89 **(b)** region inside B but outside of A shaded

Question 13

(a) The common error was not to order the numbers before locating the median. Once the numbers were ordered the common wrong answer was 0. Given that the question indicated that there were 12 numbers in the list, candidates should have been aware that the median would be the value between the middle two of the given numbers, in this case 0 and 1.

(b) The majority of candidates knew the correct method to find the mean but there were often arithmetic errors in the calculations. As the calculation produced a fraction of $\dfrac{8}{12}$, any conversion to a decimal should have had sufficient decimal places to show the recurring decimal. The mean value of the temperatures was a fraction/decimal and should not have been rounded to the nearest whole number as temperature is continuous data. Some answers of $\dfrac{8}{12}$ were incorrectly converted to 1.5.

(c) This part was answered well.

Answers: **(a)** 0.5 **(b)** $\dfrac{2}{3}$ **(c)** 8

Question 14

(a) The common error was not to use the two triangles, with corresponding sides of four and six, but to use the ratio 4:2, resulting in wrong answers of 3.6 cm and 0.9 cm.

(b) There were few candidates who recognised that the ratio of areas of similar figures is equal to the square of the ratio of corresponding sides, enabling them to give

$$\frac{\text{Area triangle APQ}}{\text{Area triangle ABC}} = \left(\frac{4}{6}\right)^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}, \text{ thus giving } \frac{\text{Area APQ}}{\text{Area PBCQ}} = \frac{4}{(9-4)} = \frac{4}{5}.$$

There were many attempts to calculate different areas, including trying to use Pythagoras' theorem. A significant number of candidates did not attempt this part.

Answers: **(a)** 2.7 **(b)** $\frac{4}{5}$

Question 15

(a) The transformation was often correctly answered although a few candidates gave more than one transformation even though the question specified only one. The common wrong transformation was translation. Qualifying the transformation fully was rare, with many candidates recognising a 90° clockwise rotation but failing to indicate that the centre of rotation was (3, 1).

(b) Candidates had difficulty dealing with a negative scale factor, with factors of 2 or $\frac{1}{2}$ sometimes being used. The centre was occasionally taken as (0, 0) or (2, 0). There were many blank responses.

Answers: **(a)** Rotation of 90° clockwise, centre (3, 1) **(b)** Triangle with vertices (–2, 4), (–4, 0), (–4, 4).

Question 16

(a) Most candidates were able to take out the common factor of five but many then did not go on to use the difference of two squares in order to complete the factorisation. Some simply 'cancelled' out the 5 to give an answer of $1 - 4t^2$ or gave $5(1 - 4t)(1 + 4t)$. A few factorised into two brackets, e.g. $(5 + 10t)(1 - 2t)$ but they then needed to continue by taking out a common factor from one bracket in order to complete the factorisation.

(b) This was generally answered well although some answers were only partially factorised and there was some confusion with signs.

Answers: **(a)** $5(1 - 2t)(1 + 2t)$ **(b)** $(3y - 2x)(y + 3)$

Question 17

(a) The common wrong answer was 77° as *ACD* was taken as 70° and not 90°.

(b) Using angles in the same segment gives the answer of 33°. The common wrong angle of 13° came from assuming that *ADE* was 90°.

(c) From the answer in **part (a)**, the answer to this part can be found by using opposite angles of cyclic quadrilateral *ABCD*.

(d) Misunderstanding of what a reflex angle is led to answers of 140° (from 70° × 2) or 35° from $\frac{70°}{2}$.

Answers: **(a)** 57° **(b)** 33° **(c)** 123° **(d)** 220°

Question 18

There were many correct answers to the simultaneous equations. Most candidates used elimination and were generally more successful that those using the substitution method. Arithmetic errors, particularly in dealing with negative signs in the elimination method and the denominators in the substitution method, often led to answers involving, e.g. sevenths for the first variable. This then often led to further errors when substituting into one of the given equations in order to find the second variable. At this point, candidates would be advised to double check their arithmetic. There were some careless arithmetic errors such as $7y = 14$ leading to $y = 7$.

Answers: $x = –4$, $y = 2$

Question 19

(a) To obtain the correct answer, candidates needed to realise that point *P* was at the end of the vector from *O* and not the vector itself. The majority of candidates were successful in answering this part.

(b) Candidates had difficulty in finding the correct position of *Q* with common errors being to mark *Q* at the end of 2**b** + **a**, at the end of 2**b** or one unit to the left or right of the correct position.

(c) This part was also difficult for candidates to visualise. The simplest way to find $\overrightarrow{OR}$ as $\overrightarrow{OQ} + \overrightarrow{QR}$, although many other routes were attempted. It was more common for –**a**, rather than –2**b**, to be correct. A number of candidates found $\overrightarrow{RO}$ rather than $\overrightarrow{OR}$. This part was often left blank.

Answers: **(a)** Point *P* marked correctly **(b)** Point *Q* marked correctly **(c)** -**a** – 2**b**

Question 20

A number of candidates did not attempt this question.

(a) Common errors were the inaccurate reading of the protractor (with answers such as 130°) and reading the wrong set of figures on the protractor (with answers such as 51°). Some candidates did not understand $D\hat{C}B$ represented an angle and gave their answer as a length.

(b)(i) Most knew the locus was an arc of a circle. The question asked for the locus **inside** the shape so the arc should have stretched from *AB* to *BC*.

(ii) Candidates had much more difficulty finding the position of the locus 5 cm from *AB*. Many simply drew arcs, of length 5 cm, with centre at *A* or at the point where their answer to part **(b)(i)** crossed *AB*, or drew random arcs of radius 5 cm.

(iii) Those drawing the two correct loci generally scored the mark in this part.

Answers: **(a)** 125° to 129° **(b)(i)** correct arc **(ii)** correct straight line **(iii)** *PD* = 3.4 cm to 3.8 cm

Question 21

(a) It was usual for partial credit to be earned here. Some final answers came from an addition or a multiplication rather than a subtraction.

(b) This part was frequently correct although errors were made in finding the determinant with values such as $\frac{1}{5}$ or $-\frac{1}{7}$. Some candidates found the inverse of another matrix seen in the working for **part (a)**.

(c) Some candidates knew the answer was the identity matrix and were able to just write it down, as asked for in the question. Others tried to carry out the multiplication, often unsuccessfully. This part was sometimes left blank.

Answers: **(a)** $\begin{pmatrix} 0 & -5 \\ 7 & 9 \end{pmatrix}$ **(b)** $\frac{1}{7}\begin{pmatrix} 3 & 1 \\ -1 & 2 \end{pmatrix}$ **(c)** $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

Question 22

(a) Most candidates failed to notice that the graph started at a speed of 4 m/s and so calculated $\frac{(8 \times 12)}{10}$, rather than working out $\frac{8 \times (12 - 4)}{10}$ and then adding 4.

(b) The simplest way to calculate the distance was to use the area of a trapezium and this was the most successful method. Many found $10 \times 12 = 120$.

(c) Few candidates knew how to draw this sketch and there was a number who did not attempt it. The first part was often drawn as a straight line rather than a concave curve. The second part was more often correct but frequently did not represent an increase of 60 m from t = 10 to t = 15. There were a significant number of single sloping straight lines drawn. Some graphs started at (0, 40) rather than (0, 0).

Answers: **(a)** 10.4 **(b)** 80 **(c)** Curve, concave upwards, from (0, 0) to (10, their **(b)**) and straight line from (10, their **(b)**) to (15, 60 + their **(b)**)

Question 23

(a) This part was answered well.

(b) There was a number of correct answers but also many of $n + 2$. A numerical answer did not fit the criteria of an expression in terms of n.

(c) Some candidates gave answers which were ythree times their answer in **part (b)** but a common error was $n + 6$ from $n + 2$. A number did not attempt this part.

(d)(i) Those spotting the pattern were generally correct.

(ii) This part was rarely correct as few could see the pattern of 3×1, 3×4, 3×9, etc. leading to the answer of $3n^2$. Clearly some schools had taught a 'method' as some candidates used the sum of an arithmetic progression for the sum of the total number of sticks, i.e. 3 + 9 + 15 + 21 + ... and others tried to use $an^2 + bn + c$ but they frequently either made arithmetic errors or did not remember that $2a = 6$. This part was often left blank.

Answers: **(a)** 7, 21 **(b)** $2n - 1$ **(c)** $6n - 3$ **(d)(i)** 48 **(ii)** $3n^2$

Question 24

(a) This part was often correct. The common errors were to give the answer as (7, 4) which was the midpoint of AB, or to subtract the coordinates, giving (4, 4).

(b) This part proved more difficult. Those who used the x with 9 and/or y with 2 often had the wrong inequality or included the equality. The answer of $x - y > 3$ was rarely correct, often due to candidates attempting to rearrange the given inequality to make y the subject, and $x - y < 3$ was common.

(c) Many candidates did not attempt this part, and those who did were generally incorrect. Although $x = 6$ and $y = 3$ satisfied $y = \frac{1}{2}x$, this point is not **inside** the triangle and fractional answers did not fit the instruction that a and b were integers.

Answers: **(a)** (9, 2) **(b)** $x < 9$, $y > 2$, $x - y > 3$ **(c)** $a = 8$, $b = 4$

MATHEMATICS D

Paper 4024/12
Paper 1

Key messages

In order to do well in this paper, candidates need to:

- have covered the whole syllabus
- remember necessary formulae and facts
- recognise, and carry out correctly, the appropriate mathematical procedures for a given situation
- perform calculations accurately
- show clearly all necessary working in the appropriate place.

General comments

It was noticeable that there were many scripts where candidates had not attempted many questions and had made poor attempts at the others. Some candidates did not appear to bring geometrical instruments into the examination.

At times, candidates appeared not to read the questions carefully. They should be encouraged to pay careful attention to the wording, the numbers given, the units used and the units required in the answer.

Questions which proved particularly difficult were **4(b)**, **7**, **11(b)**, **13**, **14(a)**, **25(c)** and **27(a)**.

It was noticeable that a significant number of candidates need to improve their ability to approximate and to use appropriate degrees of accuracy; and also to understand the integer class of number. Some candidates were very competent at performing standard techniques, and yet seemed unable to recognise the appropriate mathematical procedure required for a given situation.

Candidates need to improve their skills in manipulating fractions and in basic arithmetic, particularly when negative numbers are involved. Others need to improve their knowledge of the basic multiplication tables. Work that used correct methods was sometimes let down by incorrect calculations.

It was also noticed that some candidates need to improve their skills in solving simple algebraic equations.

Presentation of work was usually good. Candidates should bear in mind that it is to their advantage to make sure they provide sufficient working and that this working is set out neatly and legibly. Working makes it possible for marks, where they are available, to be awarded for correct methods and intermediate results.

A few candidates did not heed the instructions on the front page – to write in dark blue or black pen. Except for diagrams and graphs, candidates must not write in pencil. Nor must they overwrite pencil answers in ink, as this makes a double image which can be difficult to read. Some candidates wrote in pencil and even erased their workings, often producing rubber and paper debris that interfered with the clarity of other answers, particularly with a decimal point and a negative sign.

Candidates should be made aware that only their final answer to each question should be written in the answer space. Alternative offerings and working should not be written there. When an answer is to be changed, it is far better to delete and replace the original one rather than attempt to write over it.

Care must always be taken to ensure that answers obtained in the working are accurately transferred to the answer space.

Comments on specific questions

Question 1

(a) Generally well answered. Nearly everyone obtained the 6.573 value first. The most common error was to give 2.463 from 9.03 – 6.573.

(b) This was nearly always correct, though some, having obtained $\frac{2}{63}$, gave 31.5 as their answer.

Most candidates seemed to know the correct method, though some were not able to evaluate 8 × 7, or 9 × 6, correctly.

Answers: **(a)** 2.457 **(b)** $\frac{2}{63}$

Question 2

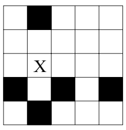
In both parts it was expected that candidates would heed the instruction to 'write down' and not embark on long methods. Each question is best solved by inspecting the positions of the decimal points and adjusting the position of the decimal point in the value that did not occur in the expression for each part. Candidates who attempted to evaluate the expressions usually made numerical, or decimal point placement errors. **Part (a)** was answered correctly more often than **part (b)**.

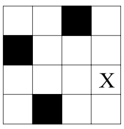
Answers: **(a)** 123.456 **(b)** 0.0643

Question 3

(a) The diagonal line of symmetry posed problems for some, but most managed to answer this part correctly.

(b) Usually answered correctly.

Answers: **(a)** 

(b) 

Question 4

(a) Generally well answered. The most common error was to evaluate 6.15/3 as 2.50. Others did not seem to notice that \$1.23 is the *total* cost of 3 pencils and gave the answer \$6.15, from 3 × \$1.23.

(b) Most candidates either did not notice the dot over the 7 in the 0.7, or else did not know that this represents a recurring 7, i.e. 0.77777... Another common error was to place –0.7 to the left of $-\frac{3}{4}$.

Answers: **(a)** 2.05 **(b)** $-\frac{3}{4}$, –0.7, 74%, $0.\dot{7}$

Question 5

(a) Most candidates knew what to do in this part. Arithmetic errors accounted for most of the wrong answers.

(b) This part showed that many candidates need to understand what is meant by a reflex angle. The preferred wrong answer was the obtuse angle 115°, from 180 – 65. Often seen was 295° from 360 – 65, with the clearly obtuse angle *BDE* assumed to be 65°.

Answers: **(a)** 41° **(b)** 245°

Question 6

This was generally well answered. The expected approximations were 2 for √3.98; 600 for 602.3; 3 for 2.987. Some struggled with the approximation for 602.3, often rounding to 6. Others used 602, from which it was possible to obtain an acceptable answer, but sometimes left their final answer as 1204/3. Very few didn't approximate the expression and tried to work out an accurate answer.

Answer: (±)400

Question 7

There were not many correct answers here. Quite often the question was not attempted, or abandoned after a dot had been placed at (3, 3).

The negative aspect to the enlargement proved problematic, possibly because the centre of enlargement wasn't the origin. There was a wide variation of incorrect answers, with the most common perhaps being an enlargement with a scale factor of +2. Some tried to enlarge from a vertex of the triangle. Others seemed to believe that the '–' implied a reduction in size, so smaller triangles were drawn.

A few obtained the points (1, 1) and (1, 5), but gave (–1, 1) as the third.

Answer: Triangle with vertices (1, 1), (1, 5) and (7, 5)

Question 8

Generally answered well by those who know what is meant by standard form. Some candidates need to learn that standard form means $A \times 10^{n}$, where $1 \leqslant A < 10$ and n is an integer.

(a) Usually well answered.
The usual wrong answers were 5.13×10^{-5}; 5.1×10^{5}; 513×10^{3}; 51.3×10^{3}.

(b) This part wasn't as well answered. Common wrong answers were 2.4×10^{-10}; 24×10^{-9}.

Answers: **(a)** 5.13×10^{5} **(b)** 2.4×10^{-8}

Question 9

Responses indicated that many candidates need to gain a better understanding of histograms.

(a) Most candidates attempted this part, though the most common answer was 10 and 50. Some gave the upper bounds of the intervals, or the widths of the time intervals.

(b) Those who answered the first part correctly usually answered this part correctly as well, though a few omitted to draw the ordinate at $t = 50$. The most common wrong answer was to draw a rectangle with a height of 3.

Answers: **(a)** 20, 25 **(b)** Rectangle with base 35 to 50 and height 2

Question 10

(a) Many candidates obtained the correct answer. However many others displayed an inability to correctly manipulate fractions or the negative numbers.

Errors that were seen included the following: $2\frac{1}{2} = \frac{3}{2}$; $4 + 3(-2\frac{1}{2}) = 4 - 6\frac{1}{2} = -2\frac{1}{2}$; $4 + (-7\frac{1}{2}) = -11\frac{1}{2}$; $4 + (-7\frac{1}{2}) = 3\frac{1}{2}$; $4\frac{1}{2}$ from $4 + 3 - 2\frac{1}{2}$; $-\frac{15}{2} = 3\frac{1}{2}$.

(b) Those who attempted this part by the direct method 5 = 4 + 3x were almost invariably successful. Many candidates attempted to find the inverse function, $\left(\frac{x-4}{3}\right)$ first, and then substitute $x = 5$, with variable success. Others merely gave 19, from $4 + 3\times5$.

Answers: **(a)** –3.5 **(b)** $\frac{1}{3}$

Question 11

(a) Most candidates made a good attempt at this part and obtained the correct answer. Others worked to the point $k = 36$ correctly, and then evaluated $\frac{36}{3^2}$ incorrectly, or evaluated $\frac{36}{3}$, or $\frac{36}{27}$.
A few decided that y varied as x, or inversely as x.

(b) Few candidates approached this part correctly. Some gave p as a function of n instead of y in terms of p. Some used $k = 36$, instead of starting with $p = kn^2$, and using $y = \frac{k}{(2n)^2}$. Very few seemed to know that with an inverse square relationship, when the independent variable is doubled, the dependent variable is divided by 2^2.

Answers: **(a)** 4 **(b)** $\frac{p}{4}$

Question 12

Some responses showed that some candidates need to get a better understanding of the different types of average.

(a) Most candidates answered this part correctly. Common wrong answers were 25; 1; 15; or an attempt to find the median or mean.

(b) Some candidates answered this part correctly. Many did not attempt to find the sum of (frequency × value), or else divided this sum by one of 2, 5, 6, 10, 15 or 25 instead of 50.

Some candidates evaluated 3×0 correctly, but gave 25 for 0×25.

A few, having obtained 40/50, gave the answer 1.25.

Answers: **(a)** 0 **(b)** 0.8

Question 13

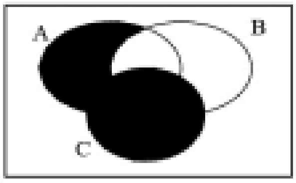
Candidates that realised, under a rotation, a point traces out an arc of a circle, centre *O*, made a good attempt by measuring the appropriate angles of 110° to intersect these arcs. Others did the equivalent by measuring the 110° angle and then the appropriate distance from *O*. Many did not seem to know what was required in this question, and offered a triangle that was a translation, or a reflection, of triangle *ABC*. A few rotated 110° anticlockwise, or by an angle other than 110°.

Answer: Correct triangle

Question 14

(a) The answers to this question were very varied and often incorrect.

(b) Those who were successful usually started with the equation $(27 – x) + x + (17 – x) + 4 = 36$, or calculated $23 + 17 – (36 – 4)$.

Answers: **(a)** 

(b) 8

Question 15

Most candidates attempted this question sensibly. There were the usual errors in forgetting to multiply both sides of an equation, performing a subtraction of one side of the equations and adding the terms on the other side, making arithmetic errors like $27 – (–8) = –35$. A very successful method was to substitute $y = 9 – 3x$ from the first equation into the second, though $3(9 – 3x)$ sometimes became $27 – 3x$.

Answer: $x = 5$, $y = –6$

Question 16

Candidates who knew the laws of indices answered this question well. Many showed that they need to improve their knowledge of these laws.

(a) Often correct. Wrong answers involved using 3^2 as 6; 3^1 as 1; 3^0 as 0, or 3; or combining the indices to give answers such as 3^3, 3^{13}, 9^3; or even 2 + 1 + 0.

(b) There were many correct answers. Common wrong answers arose from not being able to use the negative index correctly.

(c) Many candidates obtained the correct answer. Common wrong answers were $8y^3$; $4y^6$; $4y^8$; $\frac{1}{4y^3}$.

Answers: **(a)** 13 **(b)** $\frac{9}{16}$ **(c)** $4y^3$

Question 17

(a) Usually answered correctly, either by finding the three individual amounts, or directly by the calculation $\frac{60}{3}\times(5+3+2)$.

(b) Many candidates ignored the different units, or else were unable to convert $3\frac{1}{2}$ hours into minutes. Some tried to use 3.3. Others made mistakes in trying to simplify the ratio 210 : 14 by dividing 14 into 210 instead of cancelling common factors. A few tried to convert the two times into seconds, and then found it difficult to handle the large numbers obtained.

Answers: **(a)** 200 **(b)** 15 : 1

Question 18

(a) Though many candidates omitted this part, there were many reasonable attempts. Common mistakes were to write numbers in the shaded squares; calculate the products instead of the sums. A few just shaded some squares.

(b) Many realised that this event was impossible and gave the correct probability of zero. The most common wrong answer was 1.

(c) The information in the table was often used sensibly to give a probability. Many candidates gave a whole number, sometimes, but not always, corresponding to the number of squares which contained a number greater than 5.

Answers: **(a)**

-	3	4	5
3	-	5	6
4	5	-	7
5	6	7	-

(b) 0 **(c)** $\frac{1}{3}$

Question 19

Responses indicated that many candidates need to gain a better understanding of bounds.

(a) This part was quite well answered with many candidates appreciating that the lower bound is $\frac{0.1}{2}$ = 0.05 less than 1.7 . The common wrong answers were 1.2, and 1.6 .

(b) There was much less success in this part, many failing to realise that the lower bound 135 was needed. Some overlooked the 100 jars, and the conversion to kilograms. Others used 13 995.

Answers: **(a)** 1.65 **(b)** 15.15

Question 20

Responses to this question showed that most candidates realised what was required. The careless mistakes that occurred included $3(2x – 1) = 6x – 1$, or $2x – 3$; $4(x – 2) = x – 8$; $6x – 3 + 4x – 8 = 10x – 5$, $10x + 11$ or $2x – 11$; $10x – 11 = 24$ became $10x = 13$; $10x = 35$ became $x = 25$.

Answer: 3.5

Question 21

This question was poorly answered. Often candidates were not able to visualise the situation; recall the formula for the volume of a cylinder; use the formula $\frac{4}{3} \times \pi \times r^3$ with $r = 2$ for the raindrop. Those who obtained expressions for the volumes of the water in the cylinder and the volume of one raindrop sometimes subtracted these expressions instead of dividing the volume of water in the cylinder by the volume of one raindrop. Candidates should appreciate that, in questions of this type, it is not necessary to give a value to π and doing so will most likely lead to awkward calculations involving long multiplication and division. When simplifying expressions involving fractions, candidates should be encouraged to cancel rather than use long multiplication and division. Thus $6400 \times \frac{3}{32}$ becomes 200×3 by cancelling the 32s rather than obtaining $\frac{19200}{32}$ from 6400×3 and then 600 by long division.

Answer: 600

Question 22

This question was quite often omitted, perhaps because of a lack of geometrical instruments, or perhaps because many candidates need to get a better understanding of loci. Those who understand loci usually did quite well, drawing the required loci to an acceptable degree of accuracy. Occasionally there was confusion between perpendicular bisectors of sides, bisectors of angles, and medians.

Some candidates need to be made aware that, when the question requests the locus of points inside the triangle, the locus should cross the whole triangle.

(a) Many of the candidates who attempted this part gave an acceptable answer, though some did not know what was required. A few constructed a line that did not cross the whole triangle.

(b) Most of the candidates who attempted this part gave an acceptable answer. Some bisected the wrong angle, or constructed a perpendicular bisector.

(c) Those who constructed acceptable loci in **part (b)** usually shaded the correct region.

Answers: **(a)** Perpendicular bisector of *AB* **(b)** Bisector of angle *ABC*
(c) Correct (bottom right) region shaded.

Question 23

(a) There were many correct solutions seen here, particularly for those who found the area of the unshaded region, consisting of two squares and one right-angled triangle, and subtracted from 25. Those who split up the shaded region into a triangle and rectangles often made an error in finding the sides of the triangle. Some correct methods were spoilt by errors such as $1 \times 2 = 3$.

(b) Again, there were many correct solutions seen here. Many realised that the sloping side of the triangle was 5 cm long, and then counted around the perimeter. A common error was to take this length as 4 cm or calculate it to be $\sqrt{34}$. Some candidates found the perimeters of all the areas they had used in **part (a)**.

Answers: **(a)** 14 **(b)** 18

Question 24

Throughout this question problems arose when candidates assumed pairs of lines to be parallel which weren't. **Part (d)** was the most successfully answered and **part (a)** the least.

Some candidates obtained answers that they should have realised were clearly the wrong size just by looking at the diagram.

(a) This part was answered badly. Few realised that the chord *BD* subtended the angle 34° at the circumference and *x*° at the centre *O*. The usual wrong answers were 34°; 56°; 73°.

(b) Answered better than **part (a)**. Not many realised that *y*° and the 34° angle are opposite angles in the cyclic quadrilateral *ABCD* and so add up to 180°. A common wrong answer was 112°.

(c) Many candidates saw the relationship between *z* and x, or y, or the 34.

(d) Many candidates realised that angle *BAE* is an angle in a semicircle and went on to obtain the correct answer.

Answers: **(a)** 68 **(b)** 146 **(c)** 34 **(d)** 56

Question 25

(a) Although some candidates put $y = 0$ into the equation of the line, most saw that $x = 0$ should be used to find where the line meets the y-axis. There was a variety of solutions to $3y = 13$, the majority gave the correct answer to an appropriate degree of accuracy. Common wrong answers were $(4\frac{1}{3}, 0)$; (0, 4.2) and (0, 4.4), seeming to come from an attempt to read the scale. A few gave the coordinates of a point, such as (5, 1), on the line, while others read values from both axes and offered (6.4, 4.3), for example.

(b) Many candidates quoted $x \geqslant 1$ and $y \geqslant 2$, although occasionally $x \geqslant 2$ and $y \geqslant 1$ were given. A significant number did not realise that the equation of the boundary line AB was given in the question, and attempted to find it, usually without success. Perceptive candidates gave the correct inequality. A few tried to rearrange the given equation to lead to an inequality in the form $y \geqslant \ldots$ and made errors in this process.

(c) Some candidates realised what was required and usually chose one or other of the right hand corners of the shaded region. However, they did not notice that the point A lay above the line $y = 4$ and so was further from (6, 2) than (6, 6), the usual wrong answer. Other wrong answers were (5, 5); (5, 6).

Answers: **(a)** $(0, 4\frac{1}{3})$ **(b)** $x \geqslant 1$, $y \geqslant 2$, $3y + 2x \geqslant 13$ **(c)** (6, 2)

Question 26

(a) (i) There were some correct answers, but most did not attempt this part, or offered the wrong answers $n + 2$, or $2n$.

(ii) Some candidates equated their answer to the previous part, to 841, though not all could solve the resulting equation correctly. Others seemed to try a different method, or to guess.

(b) (i) The large majority of candidates who attempted the question substituted $n = 4$ into the given expression. The frequently occurring errors were to treat 4 – 4 as 1, not 0; giving the answer $6\frac{1}{2}$ from $2^3 - \frac{3}{2}$; or to evaluate $\frac{8-3}{2}$. A small number gave $4^n - \frac{(n-1)(n-4)}{4}$.

A few candidates used the expression $\frac{2^{n-1} - (n-1)(n-4)}{2}$.

(ii) Success in this part was marginally higher than in the previous part because candidates didn't have to deal with the zero in the numerator. Some candidates correctly reached $2^4 - \frac{4 \times 1}{2}$, and even $16 - \frac{4}{2}$, but went on to evaluate 8 – 4 or $16 - \frac{1}{2}$. A small number gave $5^n - \frac{(n-1)(n-4)}{5}$.

Answers: **(a)(i)** $2n - 1$ **(ii)** 421 **(b)(i)** 8 **(ii)** 14

Question 27

Attempts at this question showed that some candidates need to get a better understanding of the properties of a speed-time graph.

(a) Many candidates did not realise that the retardation is constant and that its value at $t = 10$ is the same as the gradient of the line joining (0, 30) to (20, 12). Some attempted to find, and use, the velocity when $t = 10$, but the most common wrong answer was 1.8, from (30 – 12)/10.

(b) The fact that the distance travelled can be obtained from the area under the graph was clearly known by many candidates. The quality of the arithmetic following a correct method was often the deciding factor in determining whether full, or partial, credit was gained. Common wrong answers were 180 (from $\frac{1}{2} \times 20 \times 18$); 360 (from 18×20); 600 (from 30×20); 300 (from $\frac{1}{2} \times (30 + 20) \times 12$.

(c) Successful candidates used $12(k - 20)12 = 60$, or $12x = 60$ followed by $k = 20 + x$, or $12k = (240 + 60)$. Others used $12(20 - k) = 60$ to get $k = 15$ (or an answer of 35).

Less common was $k = 3$, or even $k = 23$, (from $20k = 60$); using $12(20 + k) = 60$; trying to use proportion from 20×60/(their (b) answer).

Answers: **(a)** (–)0.9 **(b)** 420 **(c)** 25

MATHEMATICS D

Paper 4024/21
Paper 2

Key Messages

- To succeed in this paper, candidates need to have completed full syllabus coverage and remember the necessary formulae and apply them appropriately.
- Candidates should show all working clearly and use a suitable degree of accuracy in their working. Final answers should be rounded correct to three significant figures where appropriate.
- In algebraic work, candidates should take care with signs and ensure that they give their final answer in its simplest form.

General Comments

The question paper consisted of a range of question types, from routine tasks to more complex problem solving questions. This allowed candidates of all abilities to demonstrate their understanding of a range of mathematical skills. Scripts covering the whole mark range were seen and candidates appeared to have sufficient time to complete the paper. Some candidates omitted a number of the questions suggesting that they had not covered the full content of the syllabus.

In general, candidates showed good understanding of number and basic algebra work although some had difficulty in setting up and solving a linear equation. A number of candidates were unable to quote the quadratic formula correctly.

In the questions testing mensuration, many candidates were unable to distinguish between formulas for area and volume. When they were required to find the volume of a compound shape, many candidates were unable to identify all of the component parts and their dimensions which resulted in incomplete or incorrect calculations.

Candidates demonstrated good understanding of trigonometry, in particular the use of the sine and the cosine rules. If an angle in a triangle is a right angle, this will be stated in the question or indicated in the diagram: there were some cases where candidates incorrectly assumed that angles were right angles.

There were a number of places where candidates gave an inaccurate final answer due to inappropriate rounding of intermediate results. It is important that candidates retain sufficient figures in their working and only round their final answer to three significant figures. Some candidates rounded their final answers to two significant figures which is not acceptable. Values of $\frac{22}{7}$ or 3.14 were sometimes used for π and this also led to inaccurate final answers.

Comments on Specific Questions

Section A

Question 1

(a) (i) Most candidates calculated the correct monthly payment.

(ii) This question was usually answered correctly. Common errors were to leave the answer as a decimal rather than a percentage or to divide 1995 by 399 resulting in answer of 5%.

(iii) This question required candidates to identify that the price in 2016 was 105% of the price in 2015 and to carry out a reverse percentage calculation. Some candidates recognised this and then usually reached the correct answer. Many candidates interpreted the question incorrectly and calculated 95% of $1995 leading to the incorrect answer of $1895.25.

(b) Many candidates found the start time as 10 22, but in order to gain credit they needed to either give the time as 10 22 pm or 22 22 in order to identify that the time was in the evening rather than the morning. A significant number of candidates attempted to use their calculator to subtract the times and reached an incorrect result.

(c) Many candidates identified that the calculation required was 250 000 ÷ 38. Answers were not always rounded to the nearest hundred dollars as required by the question. It was common to see answers rounded to the nearest dollar or the nearest ten dollars.

(d) Most candidates knew how to calculate the speed, however few understood that they needed to calculate the bounds of the given values before carrying out the division. Those candidates who did attempt to use bounds often used the upper bound of time and distance, rather than dividing the upper bound of the distance by the lower bound of the time to give the upper bound of the speed. The calculation required was 100.5 ÷ 11.25. Some candidates did not notice that the accuracy given for the time was different from the accuracy given for the distance. A large proportion of candidates first calculated the speed using 100 ÷ 11.3 and then attempted to find an upper bound of the resulting number.

Answers: **(a)(i)** 133 **(ii)** 20 **(iii)** 1900 **(b)** 22 22 **(c)** 6600 **(d)** 8.93

Question 2

(a) Many candidates gave the correct answer. Some candidates had evaluated the expression correctly but gave an answer of 2.7 which was not acceptable because it was only given to two significant figures. Candidates who evaluated the expression in stages and rounded their intermediate values reached an inaccurate answer. Some candidates evaluated the expression as far as 19.926 but then did not find the square root of this value.

(b) Most candidates factorised the expression correctly.

(c) Many candidates expanded the brackets correctly and gave the correct answer. Some errors were seen in squaring $3a$ to give $6a^2$ or squaring b to give $2b$. A small number of candidates gave the answer $(3a + b)(3a + b)$ or $9a^2 + b^2$.

(d) Most candidates identified that they needed to use a common denominator of $(2t + 1)(3t + 1)$ and showed the correct numerator of $4(3t + 1) – 3(2t + 1)$. It was common to see a sign error in the expansion of the second bracket leading to $–6t + 3$ rather than $–6t – 3$. A small number of candidates reached the correct answer but spoilt it by incorrectly cancelling terms in the numerator and denominator.

(e) Candidates often solved the first inequality to find $n < –2.25$. Candidates should be encouraged to collect terms to reach a positive coefficient for n as those who rearranged to $–4n > 9$ usually simplified incorrectly to $n > –2.25$. Many candidates left the answer as an inequality rather than listing the integers required. Those that did give a list of integers often included –6 or –2. A sketch of a number line may have helped candidates to identify the required integers.

(f) Candidates who set up the correct equation usually solved it to reach $x = 37$. Some gave this as their answer rather than calculating the amount Chuku received using this value of x. It was common for candidates to set up the equation incorrectly, often omitting an x from the equation, using $(12 - x)$ for Bella or $(x - 12)^2$ for Chuku. Some candidates used their solution for x to calculate the corresponding amount for Chuku.

Answers: **(a)** 2.71 **(b)** $3p(3p - 2q)$ **(c)** $9a^2 + 6ab + b^2$ **(d)** $\dfrac{6t+1}{(2t+1)(3t+1)}$ **(e)** –5, –4, –3 **(f)** 50

Question 3

(a) Many candidates gave correct answers in all parts. Some candidates found correct expressions for the angles but did not write them in their simplest form as required by the question. Answers such as $180 - (90 + a)$ were common. Some candidates attempted to find numerical values for the angles rather than giving algebraic answers.

(b) (i) Candidates who used their answers to **part (a)** to identify that *APD* was an isosceles triangle generally reached the correct answer. Other candidates did not know how to use the given values to find *PB*.

(ii) Candidates who first used trigonometry to calculate the perpendicular height of the parallelogram usually found the correct area. It was common for candidates to multiply the lengths of the two sides of the parallelogram to attempt to find its area rather than using the formula base multiplied by perpendicular height. Some candidates found the area of triangle *ABD* rather than the parallelogram.

Answers: **(a)(i)** $180 - 2a$ **(ii)** $90 - a$ **(iii)** $2a$ **(iv)** $90 - a$ **(b)(i)** 3.3 **(ii)** 30.4

Question 4

(a) Many candidates identified that the radius of the inside of the bowl was 8.2 cm. As the formula for the surface area of a sphere had been given in the question, many used this formula rather than dividing it by 2 to get the surface area of the hemisphere. Some candidates used an incorrect radius of either, 8, 0.8 or 3.8.

(b) Candidates often identified that the formulas for volume of a sphere and a cylinder were required in this part, but few were able to correctly identify and combine the three volumes required. Many candidates used the formula for the volume of a cylinder to calculate the volume of the base correctly. In order to find the volume of the bowl, candidates had to subtract the volume of the inside from the volume of the outside and this was beyond all but the most able candidates. It was common for candidates to use the formula for a complete sphere rather than amending this for the volume of a hemisphere and many candidates only found one of the two volumes. Some candidates added the inner and outer volumes. Other errors were to use 3.8 or 0.8 for one of the radii of the bowl or to calculate the surface area of either the bowl or the base or both.

Answers: **(a)** 422 **(b)** 440

Question 5

(a) Many candidates answered this part correctly. Some candidates used the formula for the area of a sector rather than arc length.

(b) Many candidates did not realise that they needed to use trigonometry to calculate the lengths *OA* and *OB*. Those that did use trigonometry usually used the cosine rule to calculate *AB* and then used the fact that *OAB* was an equilateral triangle to reach *OA* and *OB*. Very few candidates drew in the line *OC* and found *OA* and *OB* using the triangles formed. Many candidates assumed that *OA* and *OB* were also 1.8 cm. Some candidates assumed that this part required them to calculate the area of the sector as they had calculated the arc length in the previous part.

(c) (i) It was very common for candidates to state that $1.8 \times 3 = 5.4$ in this part with no further justification, which was unacceptable. Few candidates identified that $OC = \dfrac{1.8}{\cos 60}$, $CT = 1.8$ and that $OC + CT$ was the radius of the semicircle. An alternative method used by some candidates was to use Pythagoras's Theorem in triangle *OBC* with the value of *OB* they had found in part (b).

(ii) Some candidates realised that *BQ* could be found by subtracting the length of *OB* from the radius of the semicircle and gave an answer that correctly followed from their *OB*. It was very common, however for candidates to assume that *BQ* was half of *OQ*.

Answers: **(a)** 3.77 **(b)** 10.0 **(c)(i)** 5.4 correctly derived **(ii)** 2.28

Question 6

(a) Most candidates identified that they could find the length using Pythagoras's Theorem and evaluated it correctly. Some candidates used a long method and found *BT* first in triangle *BCT* and then *AT* in triangle *BAT* rather than using the direct calculation from triangle *CDT*.

(b) The question identified the four triangles required for the surface area of the roof and most candidates used the correct formula for the area of a triangle and summed the areas of four triangles. Many candidates did not work out length *BT*, and assumed that it was one of the given lengths and some did not use the length of *DT* that they had calculated in the previous part. The most common incorrect answer was 102 resulting from the calculation $2\times\left(\frac{1}{2}\times 6\times 9+\frac{1}{2}\times 6\times 8\right)$.
Some candidates also included the area of rectangle *ABCD* in their calculation.

(c) Candidates who identified that the building was made from a pyramid and a cuboid usually reached the correct total volume. It was common to see 5 used rather than 6 as the height of the pyramid. Some candidates thought that the building was a pyramid and used 11 as the height in the pyramid formula. Some candidates attempted to calculate the surface area of the cuboid.

(d) Candidates who understood the term angle of elevation often used trigonometry to reach the correct answer in this part. Only a small number of candidates calculated angle *GTH* rather than angle *GHT*. In this part some candidates used either 5 or 6 in place of 11 as the height of the triangle.

Answers: **(a)** 10.8 **(b)** 139 **(c)** 504 **(d)** 50.7

Section B

Question 7

(a) Candidates who understood that bearings are measured clockwise from the North line, and expressed using three figures, reached the correct answer in this part. The most common incorrect answer was 77°.

(b) The correct answer was common in this part. The common incorrect answer was 235 resulting from 180 + 55.

(c) Most candidates identified that the sine rule was required in this part, and if they started by finding angle *ABC* = 74°, they usually reached the correct answer. Some candidates evaluated sin 51 and sin 74 and rounded the values before completing their calculation leading to an inaccurate final answer. Some candidates who used the correct values in the sine rule were unable to rearrange it correctly to find *AB*.

(d) Many candidates identified that the cosine rule was required in this part, quoted it correctly and substituted the correct values into the formula. Some were unable to then evaluate it correctly or failed to take the square root so found DC^2 rather than *DC*. A small number of candidates assumed that *ACD* was a right angle and attempted to use Pythagoras's theorem to find *DC*.

(e) This part of the question was very challenging and many candidates were unable to identify that as *CX* was the shortest distance from *C* to *AD*, then angle *AXC* was a right angle. When this fact was used, candidates often calculated either *AX* or *CX* correctly and used this to find *DX*. A correct method for finding the time taken was often used, although candidates often had difficulty converting from hours to minutes. A number of candidates attempted to measure the diagram to find *DX*, but as the diagram was not drawn to scale, this method was not successful.

Answers: **(a)** 283 **(b)** 055 **(c)** 15.4 **(d)** 20.1 **(e)** 114 minutes

Question 8

(a) Many candidates evaluated *p* correctly. Some were unable to substitute –1.5 as the index.

(b) Candidates who drew the correct axes usually plotted the points correctly and joined them with a smooth curve. Some candidates did not use the scales given in the question.

(c) Tangents were often not drawn carefully and the point of contact was not always at $x = 2.5$ but slightly to one side of this point. When an accurate tangent was drawn, candidates often calculated the gradient correctly. Some candidates calculated (change in x) ÷ (change in y).

(d) (i) Candidates usually drew this line correctly, although some used (0.4, 0) rather than (–0.4, 0).

(ii) Candidates often found the correct equation in the required form. Candidates who used their line rather than the given coordinates to find the equation sometimes reached inaccurate values for the gradient and intercept.

(iii) Candidates generally read the *x*-coordinates of the point of intersection correctly from their graph. The most common error was reading a *y*-value from the graph and giving 0.6 in place of 0.

(iv) Many candidates did not know how to approach this part and it was often omitted. Those candidates who did attempt it usually substituted their coordinates from part **(iii)** into $2^x = Ax + B$ to find the values. If one of their values was $x = 0$, they usually reached $B = 1$ and sometimes also found a correct value for *A*. Few candidates equated the equation of the line with the equation of the curve, rearranged and compared coefficients to find the exact values of *A* and *B*.

Answers: **(a)** 0.2 **(b)** correct curve **(c)** 2.2 to 2.5 **(d)(i)** correct line **(ii)** $y = 1.5x + 0.6$ **(iii)** 0 and 3.1
(iv) $A = 2.5$, $B = 1$

Question 9

(a) Many candidates calculated the income correctly, although some left the answer in cents rather than converting it to dollars as required.

(b) Most candidates approached this part by calculating the new price as 27 cents and number of boxes sold as 182 for Tuesday. When calculating the percentage change, some divided by the new income, 49.14, rather than the initial income, 42. The more efficient method of using multipliers and evaluating 0.9 × 1.3 was seldom seen.

(c) (i) Many candidates could give a correct expression for either the price or the number of boxes sold on Wednesday. It was rare to see the correct product and a correct conversion to dollars. Some candidates converted 30 cents to 0.3 dollars, but then subtracted *y* leading to $(0.3 – y)$ rather than the correct $(0.3 – 0.01y)$ for the price in dollars. Many candidates attempted to expand their product in this part, which was not required.

(ii) Many candidates did not understand what was required in this part and it was often omitted. They were required to equate their product from **(i)** with 40 and rearrange to reach the given equation but only the most able candidates were able to do this. Many candidates were confused by the cents to dollars conversion and so were unable to form a correct equation. Some candidates gained partial credit for forming an equation using their product and correctly expanding their brackets.

(iii) Many candidates were able to solve the equation correctly, usually by factorisation. Those who used the quadratic formula made more errors, usually resulting from an incorrect quadratic formula or from errors in evaluating the discriminant.

(iv) Candidates who related this question to the correct expression from part **(i)** usually gave the correct answer, but many candidates did not know where to find the information required to answer this.

Answers: **(a)** 42 **(b)** 17 **(c)(i)** $\frac{(30-y)(140+4y)}{100}$ **(ii)** $y^2 + 5y - 50 = 0$ correctly derived **(iii)** **–**10 or 5 **(iv)** 160

Question 10

(a) (i) Most candidates were able to draw a histogram with correct width bars and a linear scale on the frequency density axis. When frequency densities were calculated they were usually correct and bars were drawn accurately, with the most common error being the final frequency density calculated as 1.5 rather than 0.75. However some candidates used the frequencies as the heights rather than calculating the frequency densities. A small number of candidates left gaps between the bars which is not acceptable in a histogram.

(ii) Many candidates identified the correct midpoints of the groups and found the correct products leading to a correct value for the mean. The most common errors were to use incorrect midpoints in place of 32.5 and 37.5 or to use group widths rather than midpoints. Some candidates found the sum of the midpoints and divided by 5, the number of groups. Candidates who reached an answer that was outside the range of the times of the runners should have identified that their answer could not be the mean of the data.

(b) (i) Many candidates identified the appropriate branches from the tree diagram and calculated the product of the probabilities correctly. A small number of candidates found the sum of the probabilities rather than the product.

(ii) In this part, many candidates found the correct answer. Some candidates selected the wrong probabilities in one pair of branches, for example finding the probability of (red, blue) and (blue, blue). Others found the probability of just (red, blue) or (blue, red). A small number of candidates found the product in place of the sum and the sum in place of the product. Some candidates miscopied probabilities from the tree diagram, leading to a final probability greater than 1.

(iii) Very few candidates were able to interpret the numerators of the fractions to identify that there were 12 red counters in the bag initially.

(iv) Many candidates omitted this part as they did not know how to use the given information to calculate the required probability. The method required candidates to identify that there were 20 beads in the bag initially, of which 6 had a yellow spot and then evaluate the probability (no spot, no spot). Some candidates gained partial credit for identifying that they needed to find the product of two fractions where both the numerator and denominator of the second fraction were one less than that of the first fraction. Many candidates referred to the probabilities from the original tree diagram and multiplied them by $\frac{1}{2}$ in their products but this was not a successful strategy.

Answers: **(a)(i)** correct histogram **(ii)** 39.4 **(b)(i)** $\frac{33}{95}$ **(ii)** $\frac{48}{95}$ **(iii)** 12 **(iv)** $\frac{91}{190}$

Question 11

(a) (i) Candidates who attempted this question were usually able to answer this part correctly. Some were unable to square (–5) correctly and reached $\sqrt{119}$ rather than $\sqrt{169}$.

(ii) (a) In order to show this result, candidates were first required to state a correct vector route for $\overrightarrow{BD}$, such as $\overrightarrow{BA}+\overrightarrow{AD}$ and then show the correct vector addition to reach the required result. The small proportion of candidates who showed this first step usually went on to correctly establish the result.

(b) In this part, most candidates attempted to solve $k - 11 = -5$, rather than $2(k - 11) = -5$ as they did not identify that the x-components of the vectors showed that AC was twice BD.

(c) As many candidates had omitted (b), they also omitted this part. Some candidates gave an answer as a vector rather than a scalar.

(b) (i) Most candidates identified that this was a reflection, although some were unable to identify the mirror line correctly. It was common to see the equation of the mirror line as $y = 0$ rather than the y-axis. Some candidates were not aware that the full description required identification of the mirror line.

(ii) (a) Candidates who attempted to multiply the correct matrices in this part usually found at least one of the pairs of coordinates correctly. The most common errors occurred in finding the vertex that $(\frac{1}{2}, 1)$ was mapped on to.

(b) Few candidates found the correct matrix for this transformation. The most common method used was to identify the matrix as $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and attempt to set up two pairs of simultaneous equations using two sets of coordinates. Even when the two pairs of equations were set up correctly, candidates were seldom able to solve them correctly.

Answers: **(a)(i)** 13 **(ii)(a)** correctly establishes $\begin{pmatrix} 6 \\ k-11 \end{pmatrix}$ **(b)** 8.5 **(c)** 4.5 **(b)(i)** reflection in y-axis;
(ii)(a) (3.5, 1), (7, 2), (8, 2) **(b)** $\begin{pmatrix} -1 & 3 \\ 0 & 1 \end{pmatrix}$

MATHEMATICS D

Paper 4024/22
Paper 2

Key messages

To succeed in this paper, candidates need to have completed full syllabus coverage and should clearly show all working. Numerical working should in general maintain 3 significant figure accuracy.

General comments

There were many well-presented scripts. Many of these clearly demonstrated that candidates had enough time to complete the whole paper. Good scripts always showed all the necessary working required in any given question. Many scripts showed no working at all thus losing the possibility of earning method marks. Some scripts contained a significant number of questions where candidates had made no response.

In many scripts, correct work was accompanied by careful setting out and handwriting that was clear and easy to read. Standards in these matters were good overall. In a laudable attempt to present neat work, there are still a number of candidates who appear to work first of all in pencil, and then over-write this in ink, without erasing the pencil. When these scripts are marked, the double writing can be very difficult to decipher.

Full syllabus coverage is clearly important when attempting paper 2. Candidates of all abilities did well when standard routines are recognised, for example, finding the roots of a quadratic equation using the formula**, question 7(b)(iii)**, or the inverse of a 2×2 matrix, **question 6(d)**. In this paper, some familiar routines are always required but within problem solving contexts that perhaps do not mention them specifically. In this paper, for example, the cosine rule was required in **question 9(a)** and Pythagoras in **question 8(b)(i).** A significant number of candidates seemed not to recognise these familiar techniques in the settings of these particular questions.

Also, different topics can usefully be linked in problem solving situations. For example in **question 1(b) and (c),** good solutions often presented the appropriate arithmetical techniques in terms of algebra. Constructing relevant equations was a sure way of dealing accurately with all the information required. Setting down ideas in this way seemed to clarify thinking and helped to deal with different aspects of a problem in the right order.

The construction of equations is an area which could usefully be developed for many candidates. The question in which this was asked for directly**, question 7(b)(ii),** proved an insurmountable challenge to many candidates. Its importance also lies in the point made above, that this technique is fundamental to problem solving in a number of topics.

It is important that questions are read carefully. For example, in **question 1(a)(i) and (ii**), candidates did not always pick up the fact that the deposit of $756 mentioned in **part (a)(i)** was also an integral part of the calculation for **part (a)(ii).** Another aspect relating to the careful reading of a question is to be clear when certain information **has not** been given. In this paper, this was particularly relevant in the responses made to **question 9(a).**

The wide variety of different types of question gave candidates of all abilities the opportunity to demonstrate what they knew. In section B, **question 7** was selected by nearly all candidates, while **question 8** was probably the least popular. There was much work in **questions 10** and **11** which showed candidates handling most of these ideas with confidence. As mentioned below, certain questions such as **question 3(e)** and **question 11(b)(ii)** were solved only by candidates at the highest level.

Comments on specific questions

Section A

Question 1

(a) (i) Most candidates understood this question and were able to express the deposit of \$756 as a percentage of the basic price by constructing the relevant fraction. The denominator of this fraction was sometimes given as \$21 000 minus \$756**,** and another common error was to present this fraction the wrong way up.

(ii) This part carried on from the previous part and required the total amount that Sayeed paid, in order to purchase the car, to be expressed as a percentage of the basic price. It was hoped that printing **total** in bold in the question would prompt candidates to include the deposit of \$756 in their calculation. Missing this out led to the common wrong answer of 105.4%. In some cases there was no multiplication by 24.

(b) There were many accurate calculations seen in this part. Successful candidates were able to process all the information given in the correct sequence. Often 127% of \$21 000 was correctly evaluated as part of the problem. A common difficulty was then seen in dealing with the deposit of \$381. The subtraction of \$381 and the division by 36 had to match the description given in the question. The correct order of events was often confused.

(c) Successful candidates realised that this was a calculation involving a reverse percentage. They were able to set out their working clearly, often using appropriate algebra. The problem then was usually correctly concluded, although sometimes, \$20 000 was given as the answer. When the 2016 price could not be found as a reverse percentage, common errors included thinking that it was either 5% or 95% of \$21 000, or even just \$21 000.

Answers: **(a)(i)** 3.6 **(ii)** 109 **(b)** 730.25 **(c)** 1000

Question 2

(a) In dealing with these fractions, successful simplifications made the immediate transition from division to multiplication. In other cases, the work presented showed misconceptions regarding cancellation, with numerous algebraic and numerical slips.

(b) Again there was difficulty with the cancellation required. The common factor of 5 in the denominator was sometimes given as the final answer. From their working, it appeared that some candidates thought this was a difference of two squares problem. Others rearranged the expression given as an equation and then attempted to solve it.

(c) There was more success in this part with the correct recognition of the difference of two squares.

(d) Only the most able candidates were able to adjust the expression as presented to reveal the common factor $(p-2)$. Others achieved a successful outcome by removing the brackets and then correctly dealing with the resulting four term expression. There were many candidates who attempted to remove the brackets from the original expression and then either returned to $(q+3)$ as part of their answer or just seemed lost as to how to proceed any further.

(e) (i) The two separate equations were often correctly written. This usually led to the correct answers. There were a significant number of candidates who forgot to change the sign of for example -1 when transferring it to the other side of the equation.

(ii) Candidates who were able to turn the original equation into a quadratic by squaring each side were usually successful. The strategy of using the solutions from the previous part and forming two simultaneous equations was also used. Sometimes only one of the previous solutions was substituted resulting in only one equation so that no further progress was possible.

Answers: **(a)** $\frac{ab}{6}$ **(b)** $\frac{1}{5}$ **(c)** $(3m-2n)(3m+2n)$ **(d)** $(p-2)(q-3)$ **(e)(i)** 2 , $-\frac{8}{5}$ **(ii)** –2 , –16

Question 3

(a) Usually well done.

(b) Well done with fairly accurate plotting of points and an acceptable drawing.

(c) Usually the instruction given in the question was followed and sensible tangents were drawn. The difference fraction to calculate the gradient was usually carefully produced, although occasionally $\frac{\Delta x}{\Delta y}$ was seen.

(d) It was expected that candidates would read the points at which their curve cut the *x*-axis. This usually led to the correct answers. There were attempts to solve the equation by other means. Needless to say, this was never successful.

In some cases, candidates did not join the points (0,0) and $(1, -0.45)$ on their curve so only one solution was given. Wrong answers seemed to indicate that some candidates did not understand what was required.

(e) (i) This part proved accessible only to a minority of candidates. If attempted, this part often contained no work for which credit could be given. It was common to see $\frac{x}{20}(x^2-10)$ equated to $x^3+10x-80$ for example, and this didn't lead anywhere.

(ii) When the equation of *L* had been found, the line was accurately drawn.

(iii) Again, with *L* in place, accurate answers were given. In a number of cases there were accurate answers given in this part without any relevant, or even any, work leading to it. These candidates received no credit here.

Answers: **(a)** 3.75 **(b)** Correct curve **(c)** 0.3 to 0.5 **(d)** 0, 3.1 **(e)(i)** $y=4-x$ **(ii)** Correct line *L* **(iii)** 3.55

Question 4

(a) (i) Most candidates noticed that they were dealing with a right angled triangle and many calculated *AD* directly as expected using the cosine ratio. Some candidates used longer methods such as finding *AB* first, followed by Pythagoras.

(ii) Again, the direct method expected was often seen. In this case, since *CD* appeared in the denominator of the usual first step, extra care was needed with transposition in order to conclude correctly.

2(b) The information given in this question led directly to the equation $\frac{1}{2}\times 3\times 5\times \sin P\hat{Q}R = 6$.

Candidates who used this approach usually arrived at the angle 53.1°. Using supplementary angles, many of these went on to include 126.9° in their answer. The variety of incorrect second answers seen included the complement of 53.1°, $(360-53.1)°$ and $-53.1°$. Candidates who realised that the original triangle could be a 3,4,5 right angled triangle usually found the angle 53.1° but no other.

Answers: **(a)(i)** 2.67 **(ii)** 4.57 **(b)** 53.1, 126.9

Question 5

(a) This question was usually attempted. Most solutions gave at least two correct pairs of angles. The letters used to identify the angles in triangle *ABT* were usually *A, B* and *T*. Occasionally, the angles were given numerical values. Only in a minority of cases were full marks obtained. Although angles *CDA* and *BTA* were usually recognised as right angles, very few reasons stated in full that these were angles between a tangent and a radius.

(b) Many candidates solved this efficiently, as expected, using similar triangles. There were a number of equally effective, but longer, solutions using the sine rule. It was possible to solve this using a ratio involving *AD,* but a common error using this approach was to take *AD* to be 7. Calculations using π were sometimes seen.

Answers: **(a)** *TAB, ATB* angle between tangent and radius of a circle, *ABT* **(b)** 2.1

Question 6

(a) Candidates dealt with this confidently. As usual, care was needed to get all four of the entries correct.

(b) Again well done.

(c) This part seemed to challenge many candidates. It was expected that evaluating $\mathbf{A}\begin{pmatrix} x \\ 2 \end{pmatrix}$ would lead to 2 linear equations, one containing only *x*, whose value could then be immediately found. These candidates either failed to arrive at $\begin{pmatrix} 2x \\ 3x+2 \end{pmatrix}$ or if they did, failed to realise that this must be the same as $\begin{pmatrix} 8 \\ 2y \end{pmatrix}$. For others, this part was as straight forward as the previous two parts.

(d) Well done by all candidates.

Answers: **(a)** $\begin{pmatrix} 4 & 4 \\ 1 & 7 \end{pmatrix}$ **(b)** $\begin{pmatrix} 2 & 4 \\ 2 & 9 \end{pmatrix}$ **(c)** 4, 7 **(d)** $\frac{1}{5}\begin{pmatrix} 3 & -2 \\ 1 & 1 \end{pmatrix}$

Section B

Question 7

(a) (i) Generally this was evaluated correctly. A common error was to leave $(-0.73)^2$ as negative.

(ii) Most candidates understood what was required here resulting in may correct rearrangements. A number of candidates substituted numerical values again in an attempt to evaluate *b*.

(b) (i) There was a varied response to this part so that overall it was not well done. Correct answers were seen and also correct statements such as $PQ \times (x + 5) = 17$, without *PQ* ever being stated explicitly as required. Common incorrect algebraic answers that were seen, included *x*, $(x + 5)$ and $x(x + 5)$. Numerical answers were seen in some cases.

(ii) There were a number of ways of forming a correct equation. Some candidates did this very efficiently and concisely. Others gave themselves more algebraic manipulation in order to simplify their equation and this often led to errors. A major problem for many candidates was adding 3 to *PQ* in order to get *AB.* Some at this stage abandoned a correct *PQ* and called it *x* instead. Sometimes the 3 appeared as a factor. There were quite a number of candidates who solved the quadratic at this stage instead of deriving it. This part of the question was sometimes omitted.

(iii) Candidates again demonstrated their familiarity with the use of the formula for the solution of a quadratic equation. There were some problems in evaluating the roots correctly, usually involving negative signs. Some candidates tried to factorise the given equation.

(iv) Candidates generally showed that the idea of perimeter was understood, although some confusion between perimeter and area was evident. In a number of cases the perimeter was given as a correct algebraic expression without being evaluated.

Answers: **(a)(i)** 1.98 **(ii)** $\sqrt{x^2 - a^2}$ **(b)(i)** $\frac{17}{x+5}$ **(ii)** $3x^2 + 15x - 85$ correctly obtained **(iii)** 3.38, –8.38 **(iv)** 20.8

Question 8

(a) (i) The formula for arc length was well known. Most candidates correctly equated this to 20 but some did not realise that this equation was required in order to calculate *R*. The transposition of the equation to find *R* was generally achieved successfully. Only those candidates who calculated *R* to 2 places of decimals completed the task fully. Some solutions used an area formula.

(ii) Again the correct formula was often remembered leading to full marks in this part.

(iii) Most of the successful attempts here involved equating the length of the major sector to the circumference of the base of the cone as expected. Some candidates achieved an equivalent result using the area of the major sector and equating this to the curved surface area of the cone. This was more complicated and thus less successful, and in effect anticipated the next part of this question. The common error here was to use πR^2 for the curved surface area instead of πrR.

Some attempts tried to use volumes.

(b) (i) This part was often well done. Candidates realised that Pythagoras was relevant here. A number of candidates used 4 as the slant height of the cone.

(ii) For those candidates who realised that this problem involved similar figures, the main error was to use the ratio 64 : 25 rather than its square root. Some candidates thought that cube roots were required.

Answers: **(a)(i)** *R* calculated to 2 decimal places and rounded to 23.9 **(ii)** 239 **(iii)** 20.7 **(b)(i)** 200 **(ii)** 2.5

Question 9

(a) The added context of bearing seemed to prevent many candidates from realising that this question was their opportunity to use the cosine rule. Those who used the cosine rule often scored full marks in this part. Some candidates used the cosine rule and then the sine rule to reach the angle that would lead to the bearing asked for. Some solutions used incorrect deductions from the diagram, such as $D\hat{C}B$ = 90°, or quadrilateral *ABCD* was cyclic so that $A\hat{B}C$ = 110°. Candidates should realise that this sort of information would have been included in the question. The majority of candidates knew which angle was the bearing even if they reached it incorrectly.

(b) Candidates handled the sine rule competently. Those who lost marks here were usually those who didn't reach the angle of 52°, probably because of incorrect work in part **(a)**.

(c) (i) This part was quite testing because no diagram was given in the question. Clearly many candidates did not need one. Others attempted diagrams that were not always helpful. Where candidates had some idea that a right angled triangle was required so that a tangent ratio could be found, there seemed to be a difficulty in appreciating that one of the sides would be 17 m.

(ii) The concept of average speed was well known and this part was well negotiated by many candidates. The main problem for others was establishing the required distance of 110 m. In this problem, those candidates who worked first of all in metres per second and then converted seemed to be more successful than those who changed to kilometres at the start.

Answers: **(a)** 326 **(b)** 92.2 **(c)(i)** 13.7 **(ii)** 16.5

Question 10

(a) (i) This was approached confidently and usually successfully.

(ii) Again, often well done. A common error was to add the factor of $\frac{1}{3}$ or $\frac{2}{3}$ to the vector expression found in the previous part thus clearly presenting an invalid expression.

(iii) Many candidates were able to identify appropriate vector paths, write them accurately in vector form, and use their results to find *OC* : *CD*. Some vector paths were more straightforward than others, so care was needed in interpreting the direction of particular vectors. Some candidates became confused with positive and negative directions at this stage.

(b) (i) Often well done. The line of symmetry could sometimes be seen on the diagram but the accurate algebraic form, $y = -x$ or its equivalent was not always given. Some candidates identified a centre, and sometimes the transformation was described as a rotation.

(ii)(a) A minority of candidates drew an accurate triangle *C.* Other than that, the closest versions to *C* were reflections in $y = 1.5$. These candidates were not able to make use of the information in the stem of this part of the question.

(b) When the previous part was incorrect, values here were mostly guesswork, often 2.

(c) Quite a few candidates seemed to know how a shear matrix was related to its scale factor, but generally, just using *k* in the right place gained no credit.

Answers: **(a)(i)** 6**b** – 3**a** **(ii)** 2**b** – **a** **(iii)** 2 : 3 **(b)(i)** Reflection $y = -x$

(ii)(a) Triangle *C* drawn with vertices (2,3), (2,2) and (5,5) **(b)** 1 **(c)** $\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$

Question 11

(a) (i)(a) Generally well understood. The Cumulative frequency curve was read accurately. The common wrong answers were 300, the value on the cumulative frequency axis, and 430, again a value on the cumulative frequency axis corresponding to 50 marks.

(b) Quite well done. Some candidates still think that the range is the difference of the values on the cumulative frequency axis. Some candidates quoted only the lower quartile, or, less frequently, the upper quartile.

(c) This part again was generally well understood. The common error here was to forget to subtract.

(ii) Often correct. Sometimes answers of more than 100 were seen!

(iii) Answers seemed to be evenly distributed between paper 1 and paper 2. The answer of Paper 1 did not always gain the mark because it was not always justified by the explanation.

It was expected that candidates would use one of the ideas illustrated by parts **(i)(a)**,**(i)(c)** or **(ii)**. For example if using **(i)(a)** it was expected that the value of the median for paper 2 would be stated so that if the reason was "a lower median" then this was clearly justified.

Similarly, appropriate comparative readings were expected if using **(i)(c)** or **(ii)**. Too many explanations were vague and lacking numerical justification. Examples included steeper curve, one curve to the left of the other or one curve below the other. Some candidates thought that the question was about the examination they were taking, so that statements such as "Paper 1 because in paper 2 you can use a calculator" were seen.

(b) (i) This was well done.

(ii) Again well done.

(iii) Again, many candidates handled the combined event confidently. Some good work was let down by fraction misconceptions such as $\frac{6}{20}+\frac{6}{20}=\frac{12}{40}$.

(iv) There were quite a number of full answers. Most candidates who attempted this question recognised at least one correct combination of 3 coins and calculated the probability of this accurately.

Answers: **(a)(i)(a)** 40 **(b)** 23 to 27 **(c)** 225 to 245 **(ii)** 80 **(iii)** Paper 1 **(b)(i)** $\frac{2}{4}$ **(ii)** $\frac{2}{20}$ **(iii)** $\frac{12}{20}$ **(iv)** $\frac{18}{60}$